Week 10 Section (6/2/21)

Graph theory has a lot of definitions. The best way to learn the definitions is to do problems.

We can translate many problems into graph theory problems:

A campus department has ten faculty members $\{1,...,10\}$. As part of the department, the faculty formed committees to discuss important issues. The following committees exist:

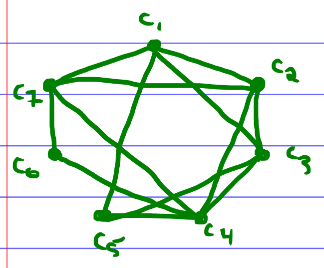
Student Engagement	$C_1=\{1,2,3\}$
Diversity and Inclusion	$C_2=\{1,3,4,5\}$
Student Disability Accessibility	$C_3=\{2,5,6,7\}$
Student Mentorship	$C_4=\{4,7,8,9\}$
Academic Advising	$C_5=\{2,6,7\}$
Undergraduate Research	$C_6=\{8,9,10\}$
Professional Development	$C_7=\{1,3,9,10\}$

The only times when faculty are all available to meet during Fall 2021 is for three hours on Friday. Is it possible for all seven committees to meet?

Vertex Set: The set of committees.

Edge Set : An edge between vertices if they share at least one member.

Step 1:



Degree c_1 : 4 *

Degree c_2 : 4

Degree c_3 : 4

Degree c_4 : 5 *

Degree c_5 : 3

Degree c_6 : 2

Degree c_7 : 4

$\{c_1, c_7\} \in E$ because $c_1 \cap c_7 = \{1,3\}$, $c_1 \cap c_2 = \{1,3\}$ $c_2 \cap c_1 =$

$c_1 \cap c_4 = \emptyset$ $\quad c_1 \cap c_3 = \{2\}$

$V = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ $\quad c_1 \cap c_5 = \{2\}$

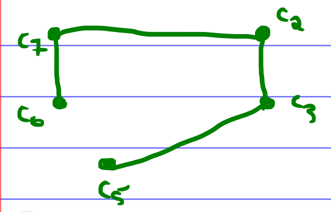
$E = \{\{c_1,c_2\},\{c_1,c_3\},\{c_1,c_5\},\{c_1,c_7\},...\}$

Timeslot 1: c_1, c_4

Timeslot 2:

Timeslot 3:

Step 2:



Degree c_2 : 2

Degree c_3 : 2

Degree c_5 : 1

Degree c_6 : 1

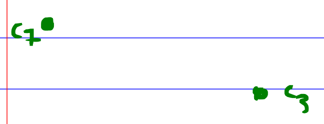
Degree c_7 : 2

Timeslot 1: c_1, c_4

Timeslot 2: c_2, c_6, c_5

Timeslot 3:

Step 3:



Degree c_3 : 0

Timeslot 1: c_1, c_4

Timeslot 2: c_2, c_6, c_5

Timeslot 3: c_3, c_7

Degree c_7 : 0

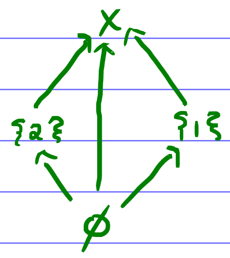
Is it possible, yes.

How many possibilities? ??

ambiguity with the term.

Recall that relations can be described by graphs. If X is a set, a relation $\sim$ on X is a subset of $X \times X$. This is a directed graph since we are taking "ordered pairs."

Ex: Let $X = \{1,2\}$, let the vertex set be the power set $\mathcal{P}(X)$ and let (A,B) be in the edge set if $A \subsetneq B$.

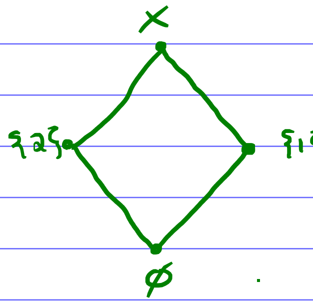


In-Degree $\{1,2\}$: 3 Out-Degree $\{1,2\}$: 0
In-Degree $\{1\}$: 1 Out-Degree $\{1\}$: 1
In-Degree $\{2\}$: 1 Out-Degree $\{2\}$: 1
In-Degree $\emptyset$: 0 Out-Degree $\emptyset$: 3

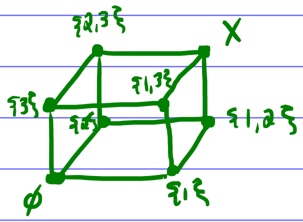
(The sum of the 'in' and 'out' degrees are same)

We can make a relation $\sim$ into an undirected graph if $\{x,y\}$ is in the edge set if $x \sim y$ or $y \sim x$.

Ex: Let $X = \{1,2\}$, let the vertex set be the power set $\mathcal{P}(X)$ and let $\{A,B\}$ be in the edge set if $A \subsetneq B$ or $B \subsetneq A$ such that there does not exist $C \in \mathcal{P}(X)$ such that $A \subsetneq C \subsetneq B$ or $B \subsetneq C \subsetneq A$.



Ex: Let $X = \{1,2,3\}$, let the vertex set be the power set $\mathcal{P}(X)$ and let $\{A,B\}$ be in the edge set if $A \subsetneq B$ or $B \subsetneq A$ such that there does not exist $C \in \mathcal{P}(X)$ such that $A \subsetneq C \subsetneq B$ or $B \subsetneq C \subsetneq A$.



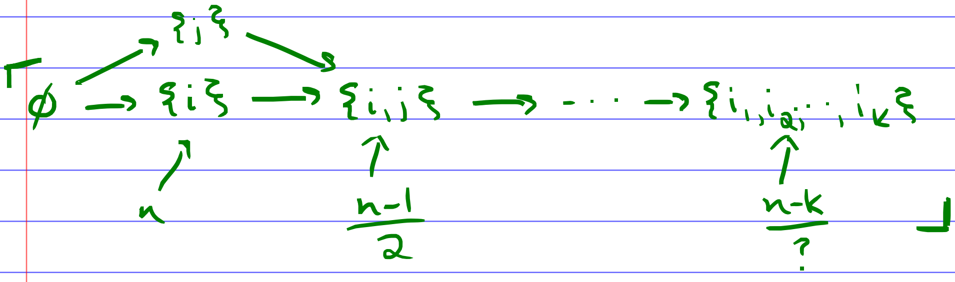
# of vertices

$2^{|X|}$

# of edges

?

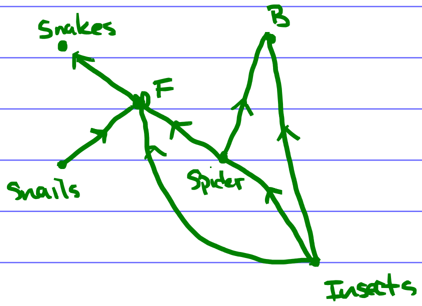
(You can use $X = \{1, \ldots, n\}$ and the edge set relation on $V = \mathcal{P}(X)$ to define an n-dimensional cube!)



Graphs can also be used to describe natural phenomena such as a food cycle in an ecosystem. Let's say in an ecosystem we have:

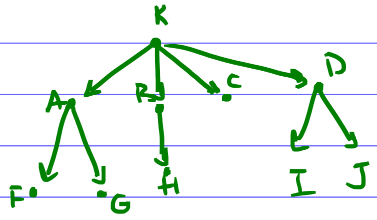
- Snakes eat frogs.
- Birds eat spiders and insects
- Spiders eat insects.
- Frogs eat snails, spiders and insects.

Vertices will be species, edges follow the transfer of energy.



Or family trees. Let vertices be people and edges will follow the lineage of a family.

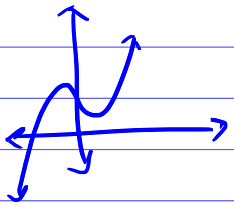
- Kyle has four children: Abigail, Brad, Chuck, Dean
- Abigail has two children: Fred, Grey
- Brad has one child: Heather
- Dean has two children: Isaac, John



$f(n) =$

Rational Root Thm

Approximate Using Derivatives.



$c(t) = t^3 - 4t^2 - 4t - 5$

$c'(t) = 3t^2 - 8t - 4$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-4)}}{2(3)} = \frac{8 \pm \sqrt{64 + 48}}{6} = \frac{8 \pm \sqrt{112}}{6}$$

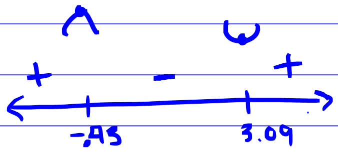
$$= \frac{8 \pm 2\sqrt{28}}{6}$$

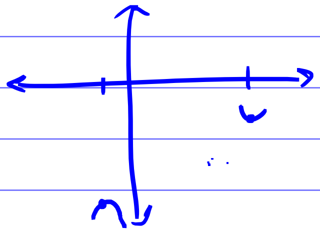
$$= \frac{8 \pm 4\sqrt{7}}{6}$$

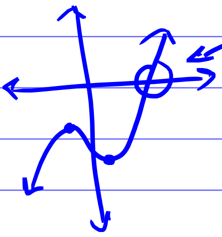
$$= \frac{4 \pm 2\sqrt{7}}{3}$$

3.09

-.43



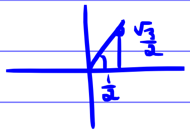




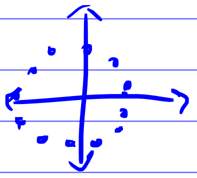
degree 1 zero

$f(n) = c_1(5)^n + c_2(-\sqrt[3]{-1})^n + c_3((-1)^{2/3})^n$

$\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^n$



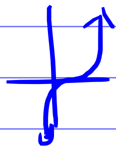
$\sqrt[n]{-1}$



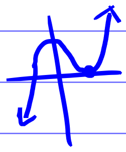
$\frac{2\pi}{n}$

$\sqrt[3]{-1} = -1$

three repeated root



two repeated



no repeated