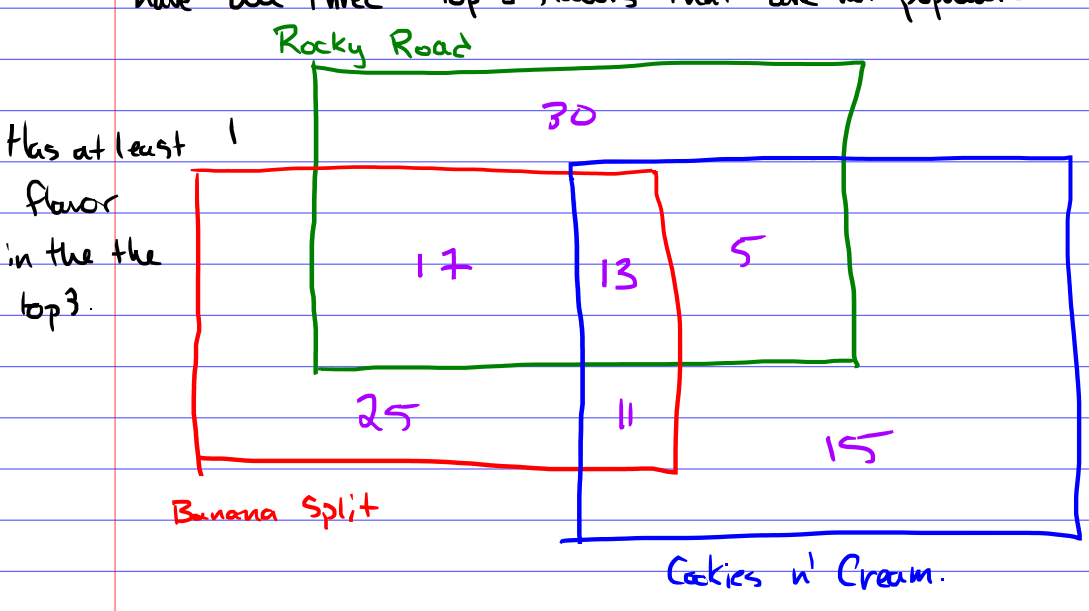
Wednesday Section (5/19/21)

An ice cream shop sent a survey to all its customers in April and recieved a 100% response rate.

- 65 respondents rated Rocky Road in their top 3.
- 66 respondents rated Banana Split in their top 3.
- 44 respondents rated Cookies n' Cream in their top 3.
- ✓ 30 respondents rated Rocky Road and Banana Split in their top 3.
- ✓ 18 respondents rated Rocky Road and Cookies n' Cream in their top 3.
- ✓ 24 respondents rated Banana Split and Cookies n' Cream in their top 3.
- ★ 13 respondents rated all three flavors in their top 3.

If 200 people responded to the survey, how many people have all three top 3 flavors that are not popular?



has at least 1 flavor in the the top 3.

116 total responses in the top 3, therefore 200 − 116 = 84 respondent have all flavor not in the top 3.

Let X be the set of people.

$A_1, A_2, A_3 \subseteq X$

RR → A_1, BS → A_2, CC → A_3

$$|X \setminus (A_1 \cup A_2 \cup A_3)| = |X| - |A_1 \cup A_2 \cup A_3|$$

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$$|X \setminus (A_1 \cup A_2 \cup A_3)| = |X| - |A_1| - |A_2| - |A_3| + |A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3| - |A_1 \cap A_2 \cap A_3|$$

"$\sum_{I \subseteq \{1,\dots,n\}} (-1)^{i} |A_I|$" where $i = |I|$

$A_\emptyset = X$

$$116 = 65 + 66 + 44 - 30 - 18 - 24 + 13$$

Generating Stirling Numbers:

First Kind

n \ k	1	2	3	4	5	6
1	1					
2	-1	1				
3	2	-3	1			
4	-6	11	-6	1		
5	24	-50	35	-10	1	
6	-120	274	-225	85	-15	1

Using
(a) $s(n,n) = 1$
(b) $s(n,k) = -(n-1)s(n-1,k) + s(n-1,k-1)$

Note that $s(n,1) = (-1)^{n-1}(n-1)!$

This follows from

$$s(k+1,1) = -k\,s(k,1) + s(k,0) = -k\,s(k,1)$$

Therefore,

$$s(n,1) = -(n-1)s(n-1,1) = (-1)^2(n-1)(n-2)s(n-2,1) \vdots = (-1)^{n-1}(n-1)\cdots 2\cdot s(1,1) = (-1)^{n-1}(n-1)!$$

By taking absolute values. $|s(n,1)| = (n-1)!$

Second kind

n \ k	1	2	3	4	5	6
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Using
(a) $S(n,n) = 1$
(b) $S(n,k) = k\,S(n-1,k) + S(n-1,k-1)$

# of partitions of a n-set into k nonempty subsets.

We can find explicit formulas for particular values of n and k. Exercise 3 has you find

- $S(n,1)$
- $S(n,2)$
- $S(n,n-1)$
- $S(n,n-2)$

Let's see $S(n,n-3)$. Let $X = \{1,\dots,n\}$, $a_i \in X$

Cases:

(1) $\{\{a_1\},\dots,\{a_{n-5}\},\{a_{n-4}\},\{a_{n-3},a_{n-2},a_{n-1},a_n\}\}$

(2) $\{\{a_1\},\dots,\{a_{n-5}\},\{a_{n-4},a_{n-3}\},\{a_{n-2},a_{n-1},a_n\}\}$

(3) $\{\{a_1\},\dots,\{a_{n-5},a_{n-4}\},\{a_{n-3},a_{n-2}\},\{a_{n-1},a_n\}\}$

$$S(n,n-3) = \underbrace{\binom{n}{4}}_{(1)} + \underbrace{\frac{\binom{n}{2}\binom{n-2}{3}}{2!}}_{(2)} + \underbrace{\frac{\binom{n}{2}\binom{n-2}{2}\binom{n-4}{2}}{3!}}_{(3)}$$