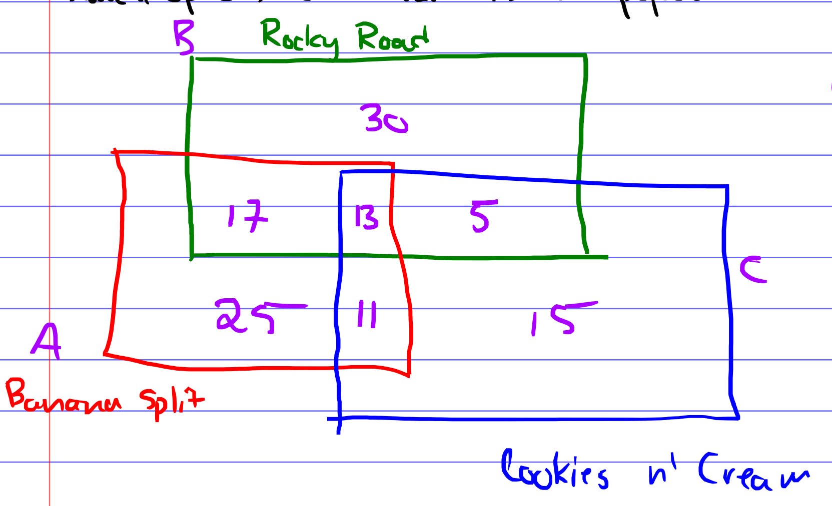
Monday Section (5/17/21)

An ice cream shop sent a survey to all its customers in April and recieved a 100% response rate.

- 65 respondents rated Rocky Road in their top 3.
- 66 respondents rated Banana split in their top 3.
- 44 respondents rated Cookies n' Cream in their top 3.
- 30 respondents rated Rocky Road and Banana Split in their top 3.
- 18 respondents rated Rocky Road and Cookies n' Cream in their top 3.
- 24 respondents rated Banana Split and Cookies n' Cream in their top 3.
- 13 respondents rated all three flavors in their top 3.

If 200 people responded to the survey, how many people have a top 3 flavor that is not popular?



$17+13+5$
35

$17+13+11$
41

$13+11+5$
29

$44 - 29 = 15$

$|A \cup B \cup C| = 30+17+13+5+25+11+15$
$= 116$

There 84 respondents where one of their top flavors was not included.

Generating Stirling Numbers:

$$\sum_{k=0}^{n} |s(n,k)| = n!$$

First Kind

n \ k	1	2	3	4	5	6
1	1					
2	-1	1				
3	2	-3	1			
4	-6	11	-6	1		
5	24	-50	35	-10	1	
6	-120	274	-225	85	-15	1

Using

(a) $s(n,n) = 1$

(b) $s(n,k) = -(n-1)s(n-1,k) + s(n-1,k-1)$

$s(n,k)$ = # of ways to write a permutation of n letters in k disjoint cycles.

Note that $s(n,1) = (-1)^{n-1}(n-1)!$

This follows from:

$s(k+1,1) = -k\,s(k,1) + s(k,0)$
$= -k\,s(k,1)$

Then, $s(n,1) = -(n-1)s(n-1,1)$
$= (-1)^2(n-1)(n-2)s(n-2,1)$
$\vdots$
$= (-1)^{n-1}(n-1)\ldots(3)(2)\cdot s(1,1)$
$= (-1)^{n-1}(n-1)!$

By taking absolute values: $|s(n,1)| = (n-1)!$

Second Kind

n \ k	1	2	3	4	5	6
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Using

(a) $S(n,n) = 1$

(c) $S(n,k) = k\,S(n-1,k) + S(n-1,k-1)$

# of ways to partition $\{1,\ldots,n\}$ into k nonempty subsets.

$\{1\} \cup \{2\} \cup \ldots \cup \{n\}$

We can find explicit formulas for particular values of n and k. Exercise 3 has you find

- $S(n,1) = 1$ $\{1,\ldots,n\}$
- $S(n,2)$
- $S(n,n-1)$
- $S(n,n-2)$

$a_i \in \{1,\ldots,n\}$

$S(n,n-3)$

★ (1) $\{\{a_1\},\{a_2\},\ldots,\{a_{n-5}\},\{a_{n-4}\},\{a_{n-3},a_{n-2},a_{n-1},a_n\}\}$

★ (2) $\{\{a_1\},\{a_2\},\ldots,\{a_{n-5}\},\{a_{n-4},a_{n-3}\},\{a_{n-2},a_{n-1},a_n\}\}$

(3) $\{\{a_1\},\{a_2\},\ldots,\{a_{n-5},a_{n-4}\},\{a_{n-3},a_{n-2}\},\{a_{n-1},a_n\}\}$

$\{\{a_1,a_2,a_3,a_4\},\{a_5\},\ldots\{a_n\}\}$

$$S(n,n-3) = \binom{n}{4} + \frac{\binom{n}{2}\binom{n-2}{3}}{2!} + \frac{\binom{n}{2}\binom{n-2}{2}\binom{n-4}{2}}{3!}$$