

HW 4 Problem 19

List the (a) partial preorders, (b) preorders, (c) partial orders, (d) orders on the set $\{1, 2, 3\}$ (let $X = \{1, 2, 3\}$)

Definition: A partial preorder is a reflexive and transitive relation.

There are a few perspectives we can take:

Note: A relation R is a subset of $X \times X$.

$$R \subseteq \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

If R is reflexive, $(1,1), (2,2), (3,3) \in R$ automatically. Thus, we are making a choice of adding these to our relation:

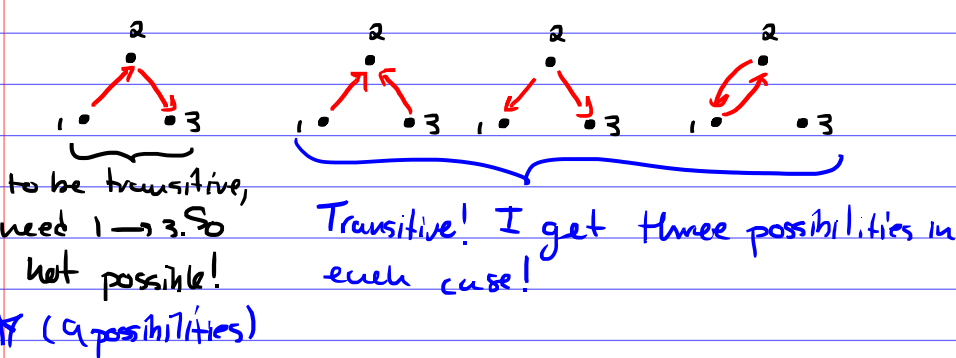
$$\{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$$

I like to think in terms of graphs, and I split into cases of how many edges I add to the relation $\{(1,1), (2,2), (3,3)\}$. In these graph, I ignore loops since they appear in all.

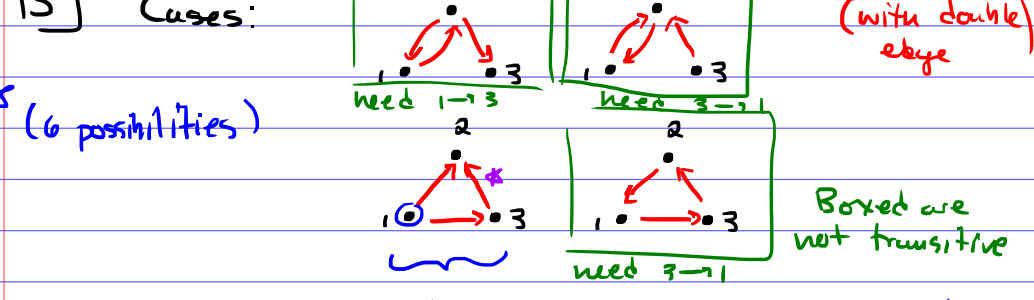
of edges added
 \square (1 possibility) \bullet Trivially transitive.

\square Consider adding any edge: (this is equivalent to adding $(1,2)$ to S). \bullet \bullet \bullet
 It doesn't matter which edge I choose, it's still transitive.
 * (6 possibilities)

\square Think of how to add to edges, we have a few cases.

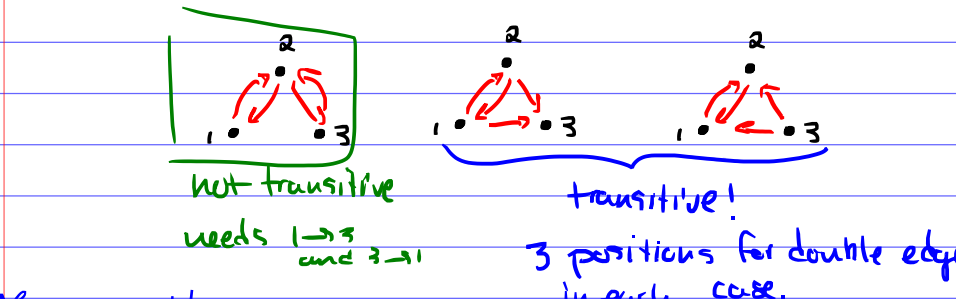


* (9 possibilities)



* (6 possibilities)
 we get 6 possibilities, fix a vertex in which two arrows start from, then two directions to choose *

\square We must have a double edge!



* 6 possibilities.

\square Two double edges required:
 but we can only add one more edge so, it will never be transitive. * 0 possibilities.

\square * 1 possibility:

29 total.

Now, you just have to list.

- 0 added $R_1 = \{(1,1), (2,2), (3,3)\}$
- 1 added
 - $R_2 = \{(1,1), (2,2), (3,3), (1,2)\}$
 - $R_3 = \{(1,1), (2,2), (3,3), (2,1)\}$
 - $R_4 = \{(1,1), (2,2), (3,3), (1,3)\}$
 - $R_5 = \{(1,1), (2,2), (3,3), (3,1)\}$
 - $R_6 = \{(1,1), (2,2), (3,3), (2,3)\}$
 - $R_7 = \{(1,1), (2,2), (3,3), (3,2)\}$
- double edge
 - $R_8 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$
 - $R_9 = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$
 - $R_{10} = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$
- same source
 - $R_{11} = \{(1,1), (2,2), (3,3), (1,2), (1,3)\}$
 - $R_{12} = \{(1,1), (2,2), (3,3), (2,1), (2,3)\}$
 - $R_{13} = \{(1,1), (2,2), (3,3), (3,1), (3,2)\}$
- same target
 - $R_{14} = \{(1,1), (2,2), (3,3), (2,1), (3,1)\}$
 - $R_{15} = \{(1,1), (2,2), (3,3), (1,2), (3,2)\}$
 - $R_{16} = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$
- choose vertex with outgoing arrows
 - $R_{17} = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,3)\}$
 - $R_{18} = \{(1,1), (2,2), (3,3), (2,1), (1,3), (2,3)\}$
 - $R_{19} = \{(1,1), (2,2), (3,3), (1,2), (3,1), (3,2)\}$
 - $R_{20} = \{(1,1), (2,2), (3,3), (1,2), (1,3), (3,2)\}$
 - $R_{21} = \{(1,1), (2,2), (3,3), (2,1), (3,1), (2,3)\}$
 - $R_{22} = \{(1,1), (2,2), (3,3), (2,1), (3,1), (3,2)\}$
 - $R_{23} = \{(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (2,3)\}$
 - $R_{24} = \{(1,1), (2,2), (3,3), (1,2), (1,3), (3,1), (3,2)\}$
 - $R_{25} = \{(1,1), (2,2), (3,3), (2,1), (3,1), (2,3), (3,2)\}$
 - $R_{26} = \{(1,1), (2,2), (3,3), (1,2), (2,1), (3,1), (3,2)\}$
 - $R_{27} = \{(1,1), (2,2), (3,3), (2,1), (1,3), (3,1), (2,3)\}$
 - $R_{28} = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,3), (3,2)\}$
- 6 edges $R_{29} = \{(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$