Midterm 2 Version B Solutions

1. Let $\lim_{x\to-4} f(x) = -7$ and $\lim_{x\to-4} g(x) = 8$. Find $\lim_{x\to-4} \frac{f(x)}{g(x)}$.
   Use limit rules,
   $$\lim_{x\to-4} \frac{f(x)}{g(x)} = \frac{\lim_{x\to-4} f(x)}{\lim_{x\to-4} g(x)} = -\frac{7}{8}$$
   Answer: B.

2. If $y = \frac{2x+3}{4x-5}$, find $\frac{dy}{dx}$.

   Use the Quotient Rule, $y' = \frac{(4x-5)2-(2x+3)4}{(4x-5)^2} = \frac{(8x-10)-(8x+12)}{(4x-5)^2} = \frac{-22}{(4x-5)^2}$.
   Answer: D.

3. Assuming that $x^4 = \cot(y)$ defines a differentiable function of x, find $\frac{dy}{dx}$ by implicit differentiation.

   Note that the derivative with respect to y of $\cot(y)$ is $-\csc^2(y)$. We can see this by rewriting $\cot(y) = \frac{cos(y)}{\sin(y)}$ and use the Quotient Rule. We take the derivative with respect to x on both sides.
   $$\frac{d}{dx}[x^4] = \frac{d}{dx}[\cot(y)]$$
   $$3x^4 = -\csc^2(y)\frac{dy}{dx}$$
   $$\frac{4x^3}{-\csc^2(y)} = \frac{dy}{dx}$$
   Answer: D.

4. If $y = x^6e^x$, find $\frac{dy}{dx}$.

   Use Product Rule, $\frac{dy}{dx} = 6x^5e^x + e^xx^6$. Each term has a e^x and x^5 in common, so we factor them out. So, $\frac{dy}{dx} = x^5e^x(6+x)$.
   Answer: C.

5. If $y = \ln(x^4 - 2x - \pi)$, find $\frac{dy}{dx}$.

   Use Chain Rule, $y' = \frac{1}{x^4-2x-\pi} \cdot (4x^3 - 2) = \frac{4x^3-2}{x^4-2x-\pi}$.
   Answer: A.

6. State whether $f(t)$ is continuous at the point $t = 5$.

$$f(t) \begin{cases} 7t - 5 & \text{if } t \neq 5 \\ -6 & \text{if } t = 5 \end{cases}$$

If I take the limit as t approaches 5, we get that $f(t)$ approaches $9 \neq -6$. So $f(t)$ is not continuous.
Answer: A.

7. Given that $f(x) = \frac{3}{x}$, find $\lim_{x \to -8} \frac{f(x) - f(-8)}{x+8}$.
This is the definition of a derivative, therefore, take the derivative of $f(x)$ and evaluate at -8, $f'(x) = -\frac{3}{x^2}$. At -8, $f'(-8) = -\frac{3}{64}$.
Answer: C.

8. If $y = x^7 \ln(x)$, find $\frac{dy}{dx}$. Use the Product Rule, $y' = 7x^6 \ln(x) + \frac{1}{x} \cdot x^7 = 7x^6 \ln(x) + x^6$.
Answer: A.

9. Determine whether $f(x) = \frac{x^2-4}{x-2}$ is continuous at $c = 2$. If the function is not continuous, determine whether the discontinuity is removable or nonremovable.

   Directly substituting 2 into $f(x)$ gives us an undefined value (dividing by zero). Therefore, it is not continuous. Factor the top, $f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x + 2$. Since the denominator cancels with a term in numerator, this discontinuity is removable, meaning there is a hole at 6. If the hole was not there, then $f(2) = 4$ (we can see this by plugging in 2 into the reduced form).
Answer: C.

10.

$$\lim_{t \to 5^+} \frac{t^2}{25 - t^2}$$

If we approach from the right $5^2 - t^2 < 0$ since $t > 5$. As t approaches 5, the denominator becomes smaller and smaller, making the value of the function become large. So the limit is $-\infty$.
Answer: D.

11. Find an equation for the line tangent to $y = \frac{27}{x^2+2}$ at $(1, 9)$.

    To find the slope of the tangent line, take the derivative and evaluate at 1. Use Chain Rule $y' = 27(-\frac{1}{(x^2+2)}) \cdot 2x = \frac{-54x}{(x^2+2)^2}$. At 1, $y'(1) = \frac{-54}{9} = -6$. To find y-intercept, plug into point-slope form.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 9 &= -6(x - 1) \\
y &= -6x + 15
\end{aligned}
$$

    Answer: C.

12. Let $\lim_{x \to 6} f(x) = 243$ and $\lim_{x \to 6} g(x) = 5$. Find $\lim_{x \to 6} \sqrt[5]{f(x)}[g(x) + 1]$.
    Use limit rules,

$$
\lim_{x \to 6} \sqrt[5]{f(x)}[g(x) + 1] = \sqrt[5]{\lim_{x \to 6} f(x)}[\lim_{x \to 6} g(x) + \lim_{x \to 6} 1] = \sqrt[5]{243}[5 + 1] = 18
$$

    Answer: D.

13.

$$
\lim_{x \to \infty} \frac{3x^3 + 1}{11x^2 - \pi x^3}
$$

    Multiply top and bottom by $\frac{1}{x^3}$, then

$$
\lim_{x \to \infty} \frac{3x^3 + 1}{11x^2 - \pi x^3} = \lim_{x \to \infty} \frac{3 + \frac{1}{x^3}}{\frac{11}{x} - \pi} = \frac{3 + \frac{1}{\infty}}{\frac{11}{\infty} - \pi} = \frac{3}{-\pi}
$$

    Answer: A.

14. Determine the points at which $f(x) = \frac{(x+8)(x-6)}{(x+8)(x-4)}$ is discontinuous.

    A rational function is discontinuous at holes and vertical asymptotes which occurs when the denominator is zero. Therefore, $(x + 8)(x - 4) = 0$ implies $x = -8$ or $x = 4$.
    Answer: D.

15. The $\lim_{x \to 5} \frac{x^2-25}{x-5}$ is a derivative, but of what function and at what point?

    Note that $\frac{x^2-25}{x-5} = \frac{x^2-(5)^2}{x-5}$. We are taking the inputs and squaring them and evaluating the limit at 5. Therefore, $f(x) = x^2$ at $x = 5$.
    Answer: C.

16. If $y = x^{5-e}$, find $\frac{dy}{dx}$.
    Use Chain Rule. $y' = (5-e)x^{5-e-1} = (5-e)x^{4-e}$.
    Answer: D.

17. Find an equation for the line tangent to $y = x + \frac{1}{x}$, when $x = 3$.

    To find the slope of the tangent line, take the derivative and evaluate at 3, $y' = 1 - \frac{1}{x^2}$. At 3, $y'(3) = 1 - \frac{1}{9} = \frac{8}{9}$. We need the corresponding y-value, so we plug into the original equation. $y(3) = 3 + \frac{1}{3} = \frac{10}{3}$. Now plug into the point-slope form and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{10}{3} &= \frac{8}{9}(x-3) \\ y &= \frac{8}{9}x - \frac{8}{3} + \frac{10}{3} \\ y &= \frac{8}{9}x + \frac{2}{3} \end{aligned}$$

    Answer: B.

18.

$$\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2}$$

    Factor then evaluate the limit.

$$\lim_{x\to 1} \frac{x^2-1}{x^2-3x+2} = \lim_{x\to 1} \frac{(x+1)(x-1)}{(x-2)(x-1)} = \lim_{x\to 1} \frac{x+1}{x-2} = \frac{2}{-1} = -2$$

    Answer: B.

19. If $y = e^{2x^2+7x}$, find $\frac{dy}{dx}$.

    Use Chain Rule, $e^{2x^2+7x} \cdot (4x+7)$
    Answer: C.

20.

$$\lim_{x\to 4^-} \frac{x-4}{|x-4|}$$

    When $x < 4$, $x - 4 < 0$, so the absolute value outputs the negative of the input.

$$\lim_{x\to 4^-} \frac{x-4}{|x-4|} = \lim_{x\to 4^-} \frac{x-4}{-(x-4)} = \lim_{x\to 4^-} -1 = -1$$

    Answer: B.