

Quiz 6

1. Find the linear approximation at $x = 4$ of $f(x) = \ln(x - 3)$.
2. Find the derivative of the following functions:
 - (a) $y = \log_3(2^x)$
 - (b) $y = \arctan(x^2)$
 - (c) $y = x^{2x}$

Solutions to Quiz 6.

1. The linear approximation is given by $L(x) = f(a) + f'(a)(x - a)$. We compute $f(a)$ and $f'(a)$. Note that $f'(x) = \frac{1}{x-3}$. Therefore, $f(4) = 0$ and $f'(4) = 1$. Then, the linear approximation is given by

$$L(x) = 0 + 1(x - 4) = x - 4$$

2. Each of these derivatives require the Chain Rule.

- (a) $y' = \frac{1}{\ln(3)2^x} \cdot (\ln(2)2^x) = \frac{\ln(2)}{\ln(3)}$.

Alternatively, simplify $y = x \log_3(2)$ and $y' = \log_3(2) = \frac{\ln(2)}{\ln(3)}$.

- (b) $y' = \frac{1}{1+(x^2)^2} \cdot (2x) = \frac{2x}{1+x^4}$.

- (c) We can use logarithms to make the derivative easier to compute:

$$y = x^{2x}$$

$$\ln(y) = \ln(x^{2x})$$

$$\ln(y) = 2x \ln(x)$$

$$\frac{y'}{y} = 2 \ln(x) + 2x \frac{1}{x}$$

$$\frac{y'}{y} = 2 \ln(x) + 2$$

$$y' = y(2 \ln(x) + 2)$$

$$y' = x^{2x}(2 \ln(x) + 2)$$