Implicit Differentiation

1. Find $\frac{dy}{dt}$ when
$$y^2 + t^2 = 9$$

   Solution 1. We will take the derivative with respect to t,

$$\begin{aligned}
\frac{d}{dt}(y^2 + t^2) &= \frac{d}{dt}(9) \\
2y\frac{dy}{dt} + 2t &= 0 \\
2y\frac{dy}{dt} &= -2t \\
\frac{dy}{dt} &= -\frac{t}{y}
\end{aligned}$$

2. Find $\frac{dx}{dt}$ when
$$xt + x^2 + 3t = t^2$$

   Solution 2. We will take the derivative with respect to t,

$$\begin{aligned}
\frac{d}{dt}(xt + x^2 + 3t) &= \frac{d}{dt}(t^2) \\
[x + t\frac{dx}{dt}] + 2x\frac{dx}{dt} + 3 &= 2t \\
x + t\frac{dx}{dt} + 2x\frac{dx}{dt} + 3 &= 2t \\
t\frac{dx}{dt} + 2x\frac{dx}{dt} &= 2t - 3 - x \\
\frac{dx}{dt}(t + 2x) &= 2t - 3 - x \\
\frac{dx}{dt} &= \frac{2t - 3 - x}{t + 2x}
\end{aligned}$$

3. Find $\frac{dy}{dx}$ when
$$y = x^2y + e^x$$

Solution 3. We will take the derivative with respect to x,

$$\begin{aligned}
\frac{d}{dx}(y) &= \frac{d}{dx}(x^2y + e^x) \\
\frac{dy}{dx} &= [x^2\frac{dy}{dx} + 2xy] + e^x \\
\frac{dy}{dx} &= x^2\frac{dy}{dx} + 2xy + e^x \\
\frac{dy}{dx} - x^2\frac{dy}{dx} &= 2xy + e^x \\
\frac{dy}{dx}(1 - x^2) &= 2xy + e^x \\
\frac{dy}{dx} &= \frac{2xy + e^x}{1 - x^2}
\end{aligned}$$

4. Find $\frac{dy}{ds}$ when

$$s^2 + s + 1 = y^2 + y + 1$$

Solution 4. We will take the derivative with respect to s,

$$\begin{aligned}
\frac{d}{ds}(s^2 + s + 1) &= \frac{d}{ds}(y^2 + y + 1) \\
2s + 1 &= 2y\frac{dy}{ds} + \frac{dy}{ds} \\
2s + 1 &= \frac{dy}{ds}(2y + 1) \\
\frac{2s + 1}{2y + 1} &= \frac{dy}{ds}
\end{aligned}$$

5. Find $\frac{dx}{ds}$ when

$$\sin(x) + xs + \sin(s) = 1$$

Solution 5. We will take the derivative with respect to s,

$$\begin{aligned}
\frac{d}{ds}(\sin(x) + xs + \sin(s)) &= \frac{d}{ds}(1) \\
\cos(x)\frac{dx}{ds} + [\frac{dx}{ds}s + x] + \cos(s) &= 0 \\
\cos(x)\frac{dx}{ds} + \frac{dx}{ds}s + x + \cos(s) &= 0 \\
\cos(x)\frac{dx}{ds} + \frac{dx}{ds}s &= -\cos(s) - x \\
\frac{dx}{ds}(\cos(x) + s) &= -\cos(s) - x \\
\frac{dx}{ds} &= \frac{-\cos(s) - x}{\cos(x) + s}
\end{aligned}$$