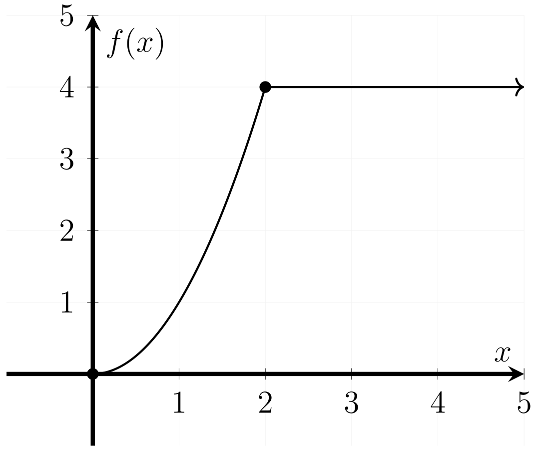
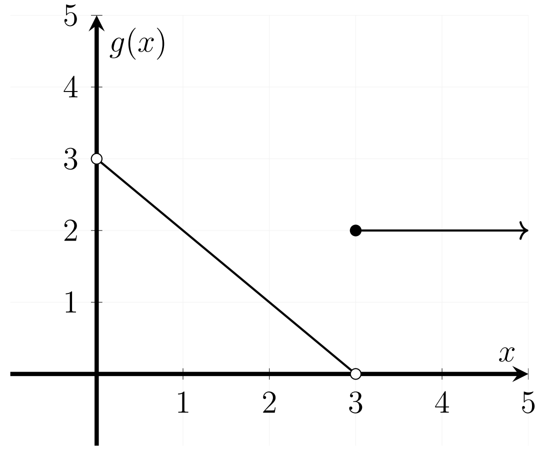
Quiz 4

Use the following graphs of the functions f and g for question 1,





1. Compute the following limits:

   (a) $$\lim_{x\to 3^+} f(x)+g(x)$$

   (b) $$\lim_{x\to 3^-} \frac{f(x)}{g(x)}$$

   (c) $$\lim_{x\to 2} g(x)-f(x)$$

   (d) $$\lim_{x\to 1} 3f(x)g(x)$$

2. Compute
$$\lim_{x\to 0}\frac{\sin(7x)}{x}$$
3. What is the definition of a continuous function?
4. Is a polynomial continuous on the interval $[1,\infty)$?
5. What is the definition of a derivative at a point a?
6. What is the equation of the tangent line at $x=2$ of $f(x)=x^2$.

Solutions to Quiz 4.

1. Using the graphs, we compute the following limits,

   (a)
   $$\lim_{x\to 3^+} f(x) + g(x)$$
   Note that we can use the addition rule for limits and compute the limits separately. For
   $$\lim_{x\to 3^+} f(x)$$
   there are no discontinuities, therefore, we can directly substitute the value of the function. So, we get the limit is 4. For the second limit,
   $$\lim_{x\to 3^+} g(x)$$
   note that depending on which side we evaluate the limit from, we get different value. Since $x \to 3^+$, we are approaching from the right-hand side, which is 2. Now, we add the values together.
   $$\lim_{x\to 3^+} f(x) + g(x) = 4 + 2 = 6$$

   (b) Note that the left-hand limit of $g(x)$ is 0, therefore, the limit does not exist regardless of what limit of $f(x)$ is. So, by using the quotient rule of limits, we obtain that
   $$\lim_{x\to 3^-} \frac{f(x)}{g(x)} = \text{DNE}$$

   (c)
   $$\lim_{x\to 2} g(x) - f(x)$$
   Using the subtraction rule, we evaluate the limits separately. Note that
   $$\lim_{x\to 2} g(x) = 1$$
   and that
   $$\lim_{x\to 2} f(x) = 4$$
   Therefore,
   $$\lim_{x\to 2} g(x) - f(x) = 1 - 4 = -3$$

   (d)
   $$\lim_{x\to 1} 3f(x)g(x)$$

Using the product rule of limits, we evaluate the limits separately. Note that using the constant rule for limits, we will multiply our solution by 3. We obtain

$$\lim_{x \to 1} f(x) = 1$$

and

$$\lim_{x \to 1} g(x) = 2$$

Therefore,

$$\lim_{x \to 1} 3f(x)g(x) = 3(1)(2) = 6$$

2. Compute

$$\lim_{x \to 0} \frac{\sin(7x)}{x}$$

Note that

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$$

Therefore,

$$\lim_{x \to 0} \frac{\sin(7x)}{x} = \lim_{x \to 0} 7 \cdot \frac{\sin(7x)}{7x} = 7 \cdot 1 = 7$$

3. We say a function f is continuous on an interval from a to b if for all values c between a and b, $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = f(c)$.
4. A polynomial is continuous on $(-\infty, \infty)$, therefore, since $[1, \infty)$ is a subset of $(-\infty, \infty)$, then any polynomial is also continuous on $[1, \infty)$.
5. The limit definition of a derivative of a function f at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

6. To compute the equation of the tangent line, we need a slope and a y-intercept. We can use the derivative to find the slope at 2,

$$\begin{aligned} f'(2) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} \\ &= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \\ &= \lim_{h \to 0} \frac{4h + h^2}{h} \\ &= \lim_{h \to 0} 4 + h \\ &= 4 + 0 = 4 \end{aligned}$$

We need a coordinate to find the y-intercept, therefore, we plug 2 into our function, $f(2) = 2^2 = 4$. Therefore, we have the coordinate $(2, 4)$. Now, we solve for the y-intercept,

$$\begin{aligned} 4 &= 4(2) + b \\ -4 &= b \end{aligned}$$

Therefore, the equation of the tangent line is given by $y = 4x - 4$.