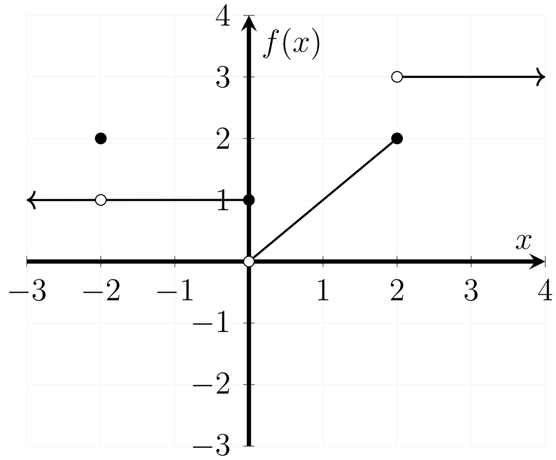
Quiz 3

Use the following graph of the function f for questions 1-4,



1. Compute
$$\lim_{x\to 2} f(x)$$

2. Compute
$$\lim_{x\to 0^+} f(x)$$

3. Compute
$$f(-2)$$

4. Compute
$$\lim_{x\to\infty} f(x)$$

5. Find the following limit, if it exists:
$$\lim_{x\to -2} \frac{x^2+3x+2}{x+2}$$

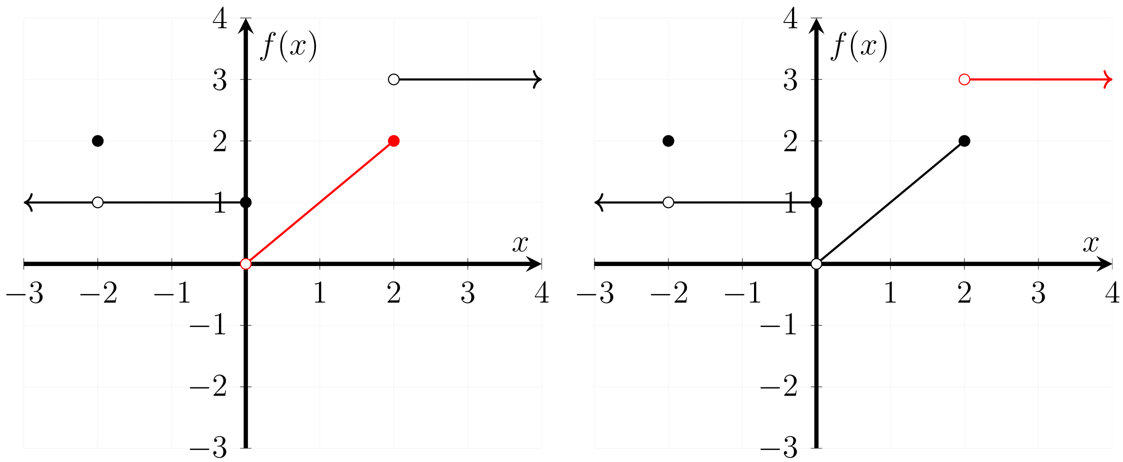
6. Determine whether or not a function exists with the following conditions:
$$\lim_{x\to 2^+} f(x) = 1 \text{ and } \lim_{x\to 2^-} f(x) = \text{DNE}$$

Solutions to Quiz 3.

1. Compute

$$\lim_{x\to 2} f(x)$$

Consider taking both one-sided limits and comparing them. On the left of $x = 2$, the function is given by $f(x) = x$ and on the right of $x = 2$, the function is given by $f(x) = 3$.



$$\lim_{x\to 2^-} f(x) = 2 \qquad \lim_{x\to 2^+} f(x) = 3$$

Therefore,

$$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$$

Therefore, the limit does not exist.

2. Compute

$$\lim_{x\to 0^+} f(x)$$

The function on the right of $x = 0$ is given by $f(x) = x$. Therefore, the $f(x) = 0$ as x approaches 0 from the right. So, the limit is 0.

$$\lim_{x\to 0^+} f(x) = 0$$

3. Compute

$$f(-2)$$

To compute this, we only need to see where the function is defined at $x = -2$, which is 2. So $f(-2) = 2$.

4. Note that the function for all values to the right of $x = 2$ is given by $f(x) = 3$. Therefore,
$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 3 = 3$$

5. If we directly substitute $x = -2$, we get $\frac{0}{0}$, which is an indeterminant form. Therefore, more work is needed to be done before evaluating. We may factor the numerator and simplify as follows:
$$\lim_{x\to-2} \frac{x^2 + 3x + 2}{x + 2} = \lim_{x\to-2} \frac{(x + 2)(x + 1)}{x + 2} = \lim_{x\to-2} x + 1 = -2 + 1 = -1$$

6. A function with these conditions does exist, consider the following piecewise function:
$$f(x) = \begin{cases} \sin(\frac{1}{x-2}) & x < 2 \\ 1 & x \geq 2 \end{cases}$$

   On the left of 2, the function oscillates as x approaches 2 and therefore, does not approach a single value, whereas on the right it is a constant function, so as x approaches 2, the function approaches 1.

   https://www.desmos.com/calculator/jkcqetpv1u