Hexagonal Mosaic Knots

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Research Experience for Undergraduates

August 28, 2017

Definition of a Knot

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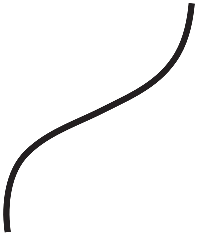

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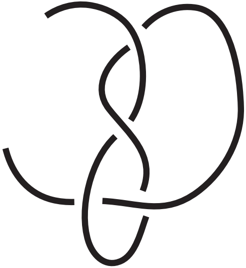

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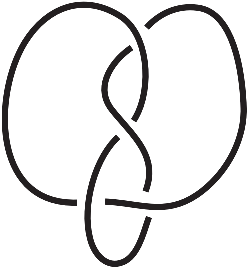
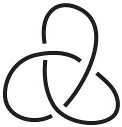
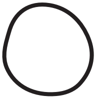


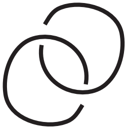
Common Terminology



Knot



Unknot



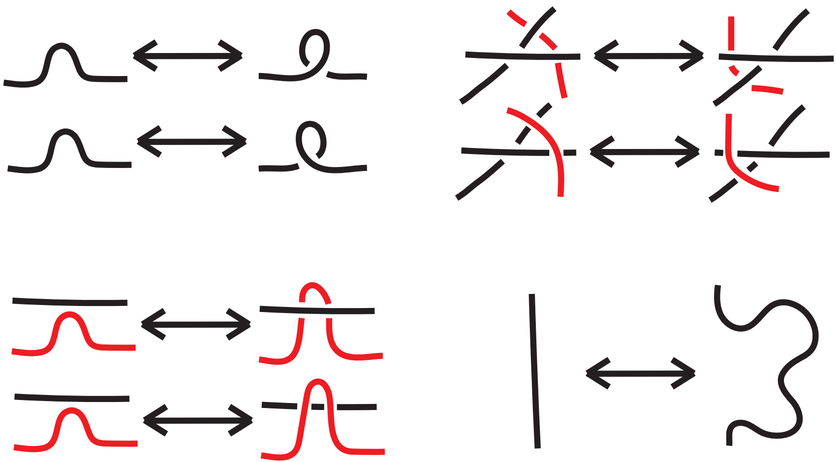
Link

Crossing

Undercrossing

Overcrossing

Equivalence of Knots



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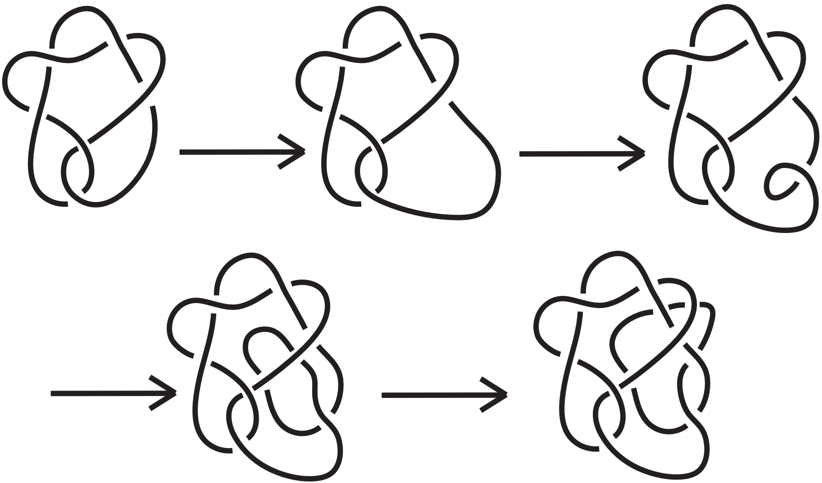
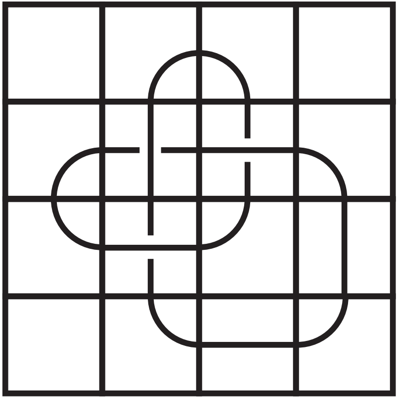


What is a Mosaic Knot?



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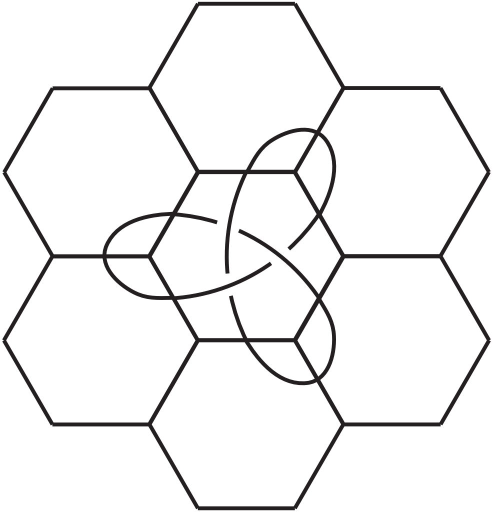


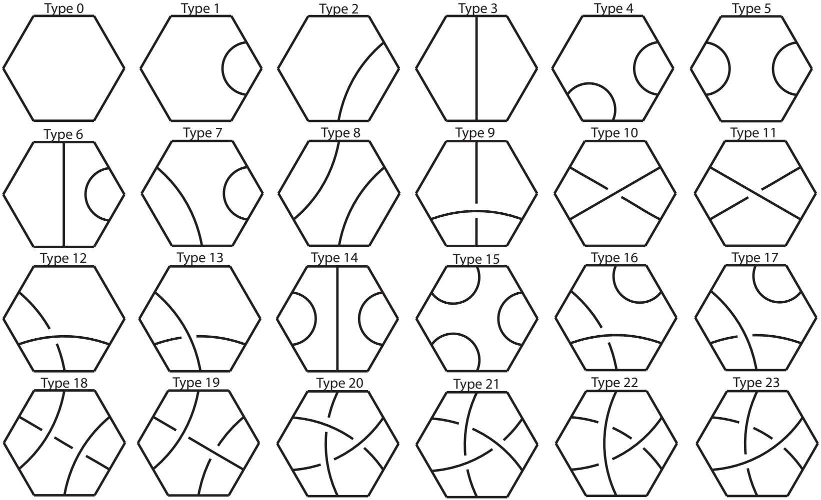
Defining the Hextile

A hextile must obey the following axioms:

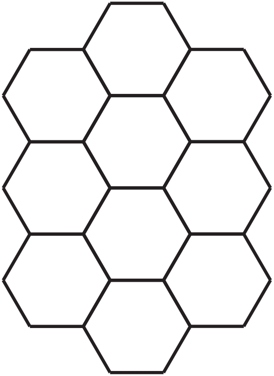
- A curve must terminate at the midpoint of an edge and a curve cannot cross itself.
- Two curves cannot cross more than once and cannot share an edge.

Examples of Violations

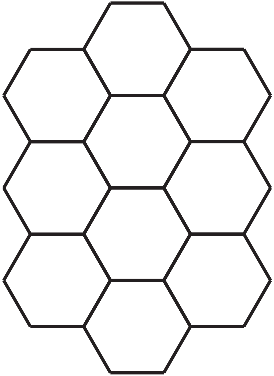
Types of Hextiles



Arrangements and Diagrams



Arrangement of ten hextiles.



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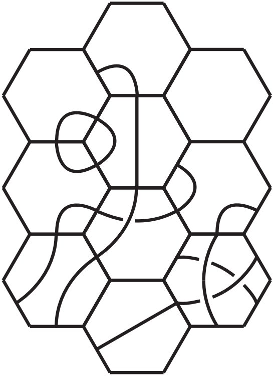
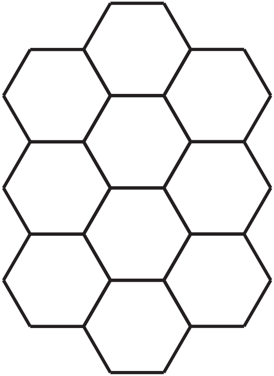


Not suitably connected.



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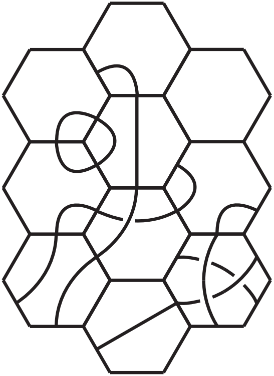

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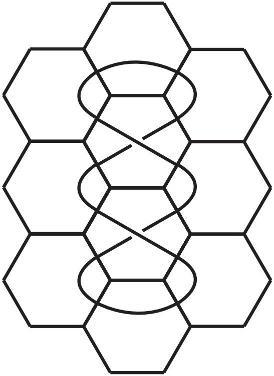
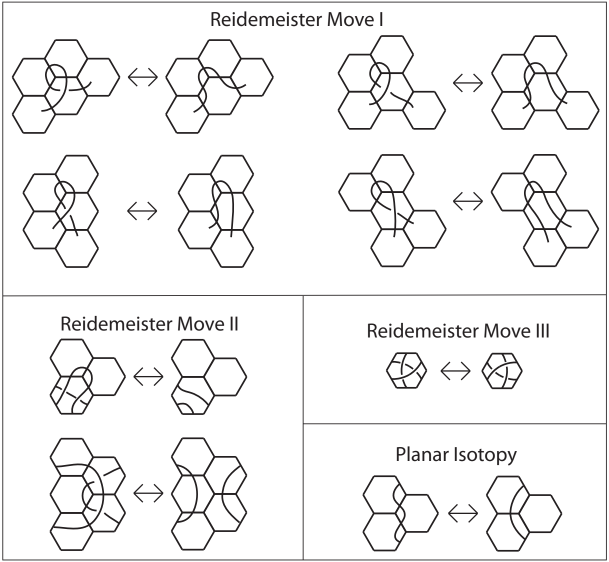


Suitably connected.

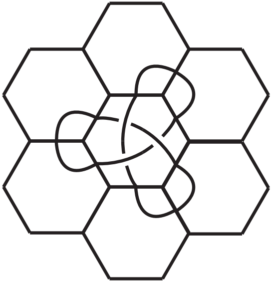
Reidemeister Moves & Planar Isotopy

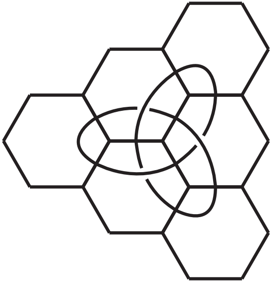


The Hextile Number

Definition

The *hextile number* of a link L is the least number of hextiles needed to represent L, denoted $h(L)$.





More crossings per tile does not imply hextile number.

Hextile Number as a Knot Invariant

Definition

The *hextile number* of a link L is the least number of hextiles needed to represent L, denoted $h(L)$.

Theorem

The hextile number is knot invariant.

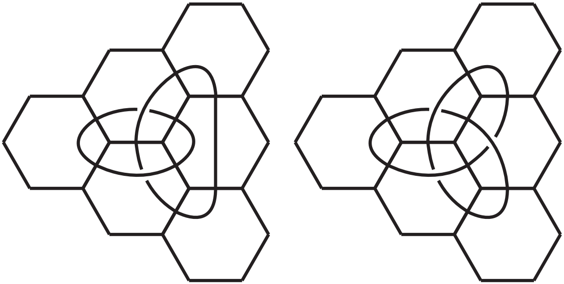
Proof.

Similar to the crossing number, given two knots if the hextile numbers are different then the knots must be different, and if the hextile numbers are the same then we can't conclude the knots are different. □

Theorems About Hextile Number

Theorem

For a non-trivial link L, $h(L) \geqslant 6$.



$h(2_1^2) = 6$ and $h(3_1) = 6$.

Proof Concept: Pincer Movement

Construction

- To show that we actually have in our hands the hextile number for some knot, our main technique has been squeezing the upper and lower bounds.
- We want to show that $h(3_1) = 6$; it is sufficient to show that $h(3_1) > 5$, and that $7 > h(3_1)$. We already have it on 6.
- Computations become exponentially harder as the number of hextiles increases.

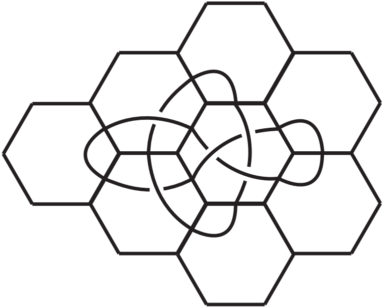
Theorems About Hextile Number and Crossing Number

Theorem

For a link L, if $c(L) \geqslant 4$, then $h(L) \geqslant 8$.

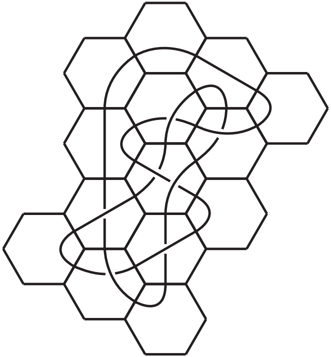
Theorem

For a knot K, if $c(K) \geqslant 5$, then $h(K) \geqslant 9$.



Known and Unknown Hextile Numbers

L	$h(L)$
0_1	3
0_1^2	5
2_1^2	6
3_1	6
4_1	8
4_1^2	8
5_1^2	8
5_1	9
5_2	9
$3_1\#3_1$	9



Is this $D(7_3)$ reducible?

L	$h(L)$
6_1	9
6_2	9
6_3	9
7_1	12?
7_2	13?
7_3	14?
7_4	11?
7_5	11?
7_6	10?
7_7	9

Thank You!

I would like to thank:

- My research mentor, Dr. Jennifer McLoud-Mann
- My faculty mentor, Dr. Alison Lynch
- The National Science Foundation Grant DMS1460699
- Undergraduate Research Opportunities Center

References

[1] ADAMS, C.
The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots.
Berlin; New York: American Mathematical Society, 2004.

[2] GALLIAN, J. A.
Contemporary Abstract Algebra.
Houghton Mifflin, Boston, New York, 2006.

[3] HOWARDS, H., AND KOBIN, A.
Crossing number bound in knot mosaics.
arXiv:1405.7683 (2014).

[4] LOMONACO, S. J., AND KAUFFMAN, L. H.
Quantum knots and mosaics.
Quantum Information Processing 7, 2-3 (2008), 85–115.

[5] LUDWIG, L. D., EVANS, E. L., AND PAAT, J. S.
An infinite family of knots whose mosaic number is realized in non-reduced projections.
Journal of Knot Theory and Its Ramifications 22, 07 (2013), 1350036.