Hexagonal Mosaic Knots

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Research Question

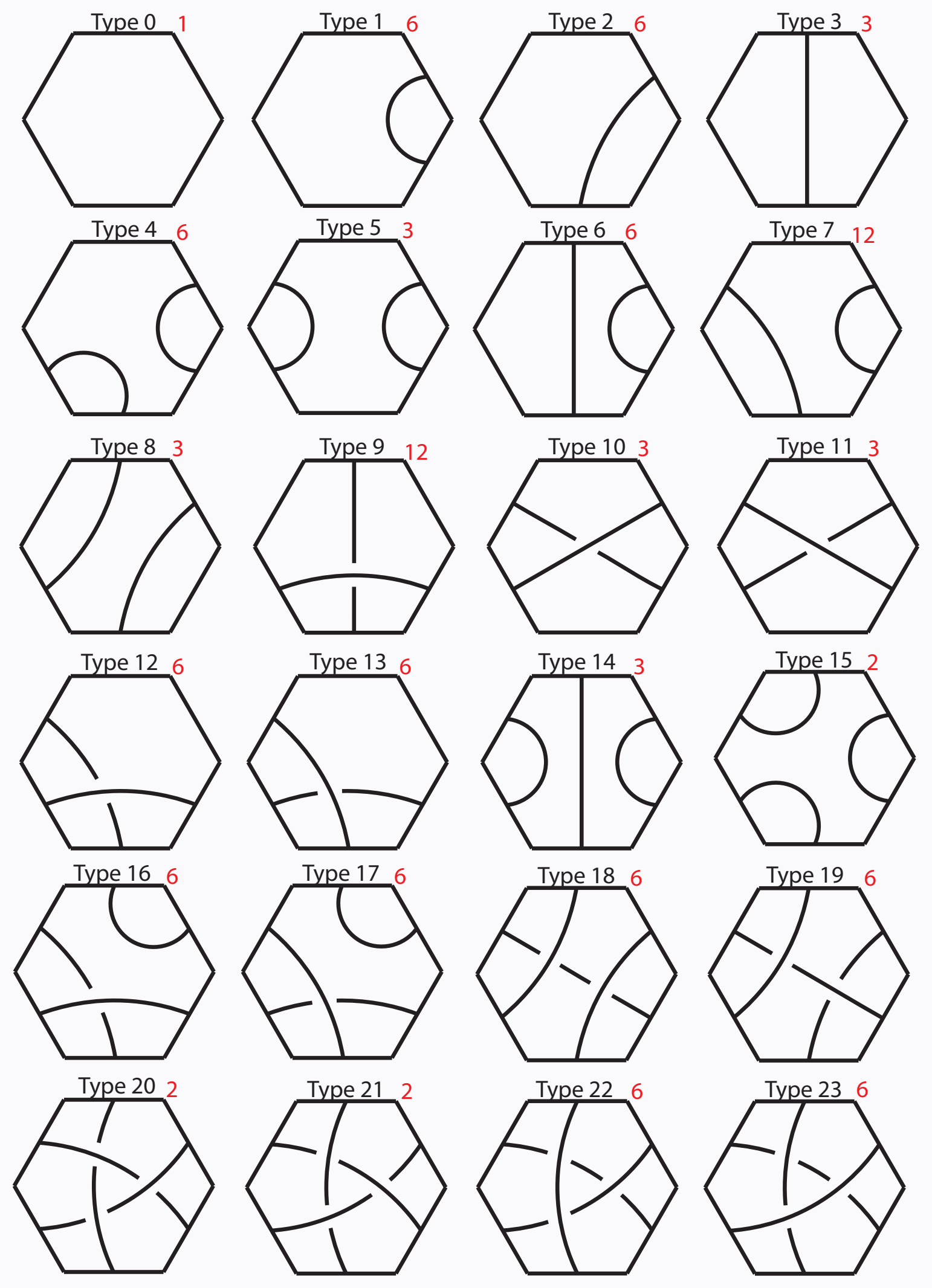
In 2008, Lomonaco and Kauffman introduced square mosaic knots in their paper *Quantum knots and mosaics* [3]. Our research considers the extension of mosaic knots from squares to hexagons and develops results for analogous arguments. We are interested in examining how hexagonal mosaic knots are similar to square mosaic knots and how are hexagonal mosaic knots are unique to mosaic knot theory.

What is a Mosaic Knot?

A mosaic knot is a knot projection on a tessellation of tiles that are placed edge to edge. We restrict our tiles to have specific properties that simplify the structure of the knot projections and create interesting relationships between the tiles and knots. In previous works, mosaic knots have been studied exclusively on square tilings, we introduce a hexagonal mosaic tile and investigate knot invariants and properties that are unique hexagonal mosaic knots.

Hextiles

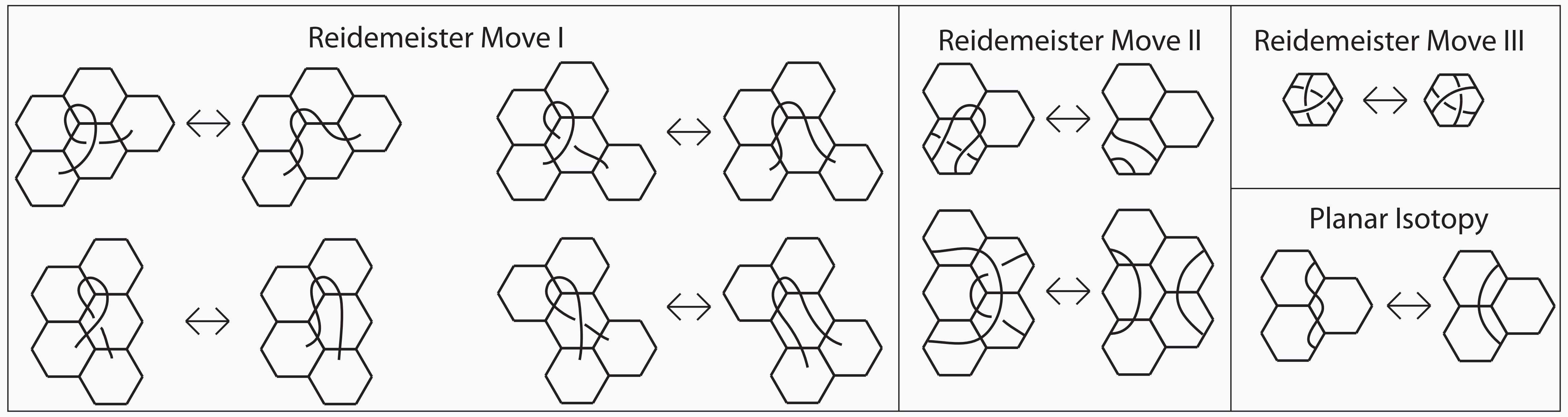
We categorize hextiles into 24 types up to rotation and reflections.



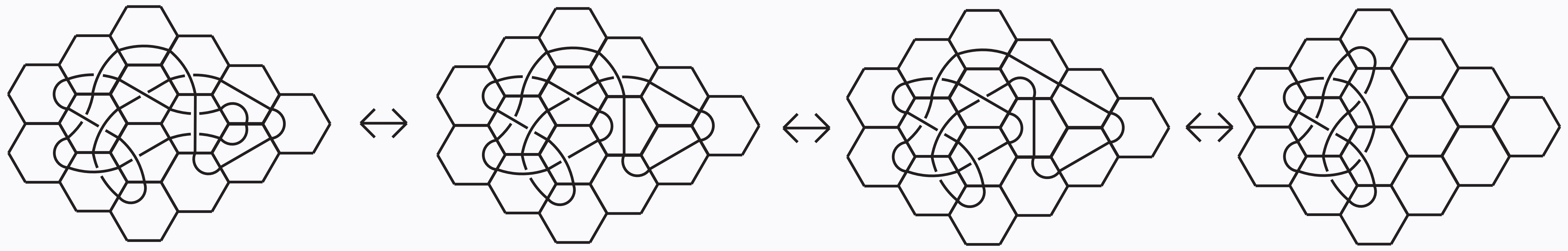
Reidemeister Moves and Planar Isotopy

What is Planar Isotopy and What are Reidemeister Moves?

Reidemeister moves allow us to move strands over each other and untwist unnecessary crossings without changing the knot. Planar isotopy allows us to stretch or compress the length of a strand without changing the knot. We adapt Reidemeister moves and planar isotopy from classical knot theory and translate them into hexagonal mosaic knot theory.



Applying Reidemeister Moves and Planar Isotopy



Using Reidemeister moves, we can reduce the amount of hextile necessary for a specific knot.

Minimal Hextile Numbers

In order to prove minimal hextile numbers, we began by creating a knot projection. We did this by taking a classical knot projection and recreating the knot on hextiles using a vector based image editor. After we created a mosaic on hextiles, we then used Reidemeister moves and planar isotopy to develop a conjecture about the minimal amount of hextiles required to create the given link. Once we had a strong conjecture, we consider the placement of crossings and showed that all other possibilities lead to larger hextile numbers than our conjecture. Our results are listed to the right.

Theorem

$h(3_1) = 6$
$h(2^2_1) = 6$
$h(4_1) = 8$
$h(4^2_1) = 8$
$h(5^2_1) = 8$
$h(5_1) = 9$
$h(5_2) = 9$
$h(6_1) = 9$
$h(6_2) = 9$
$h(6_3) = 9$
$h(3_1 \# 3_1) = 9$
$h(7_7) = 9$

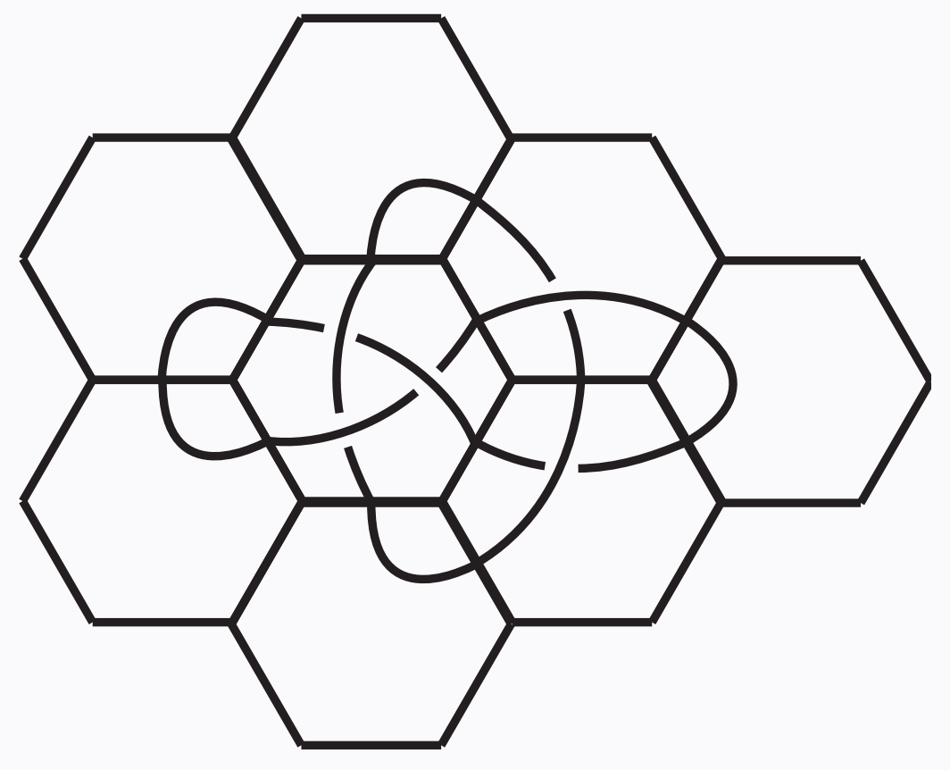


Figure 1: Whitehead Link (5^2_1)

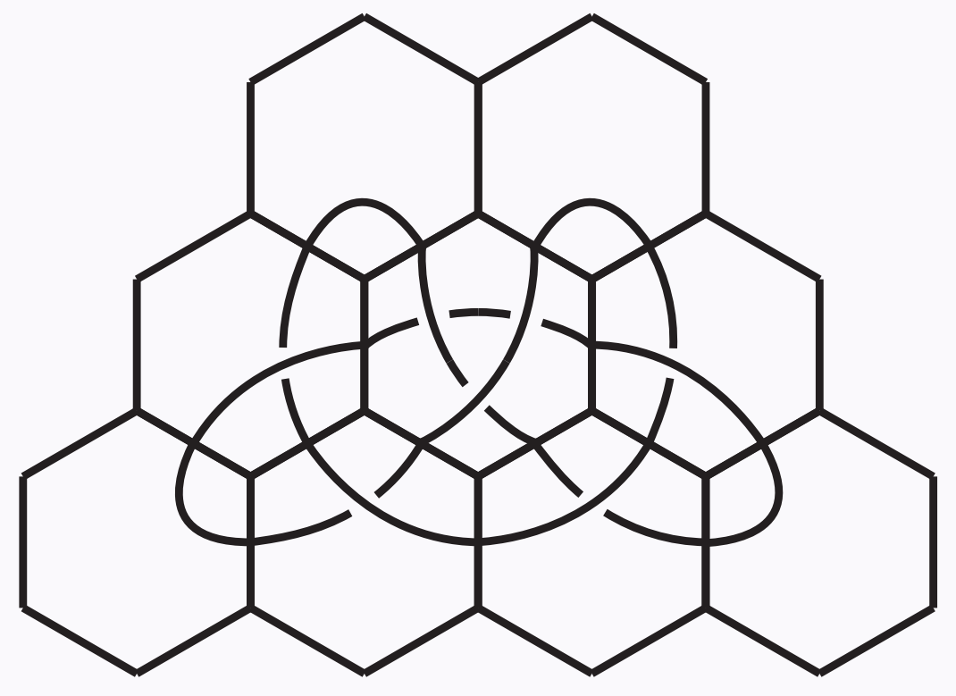
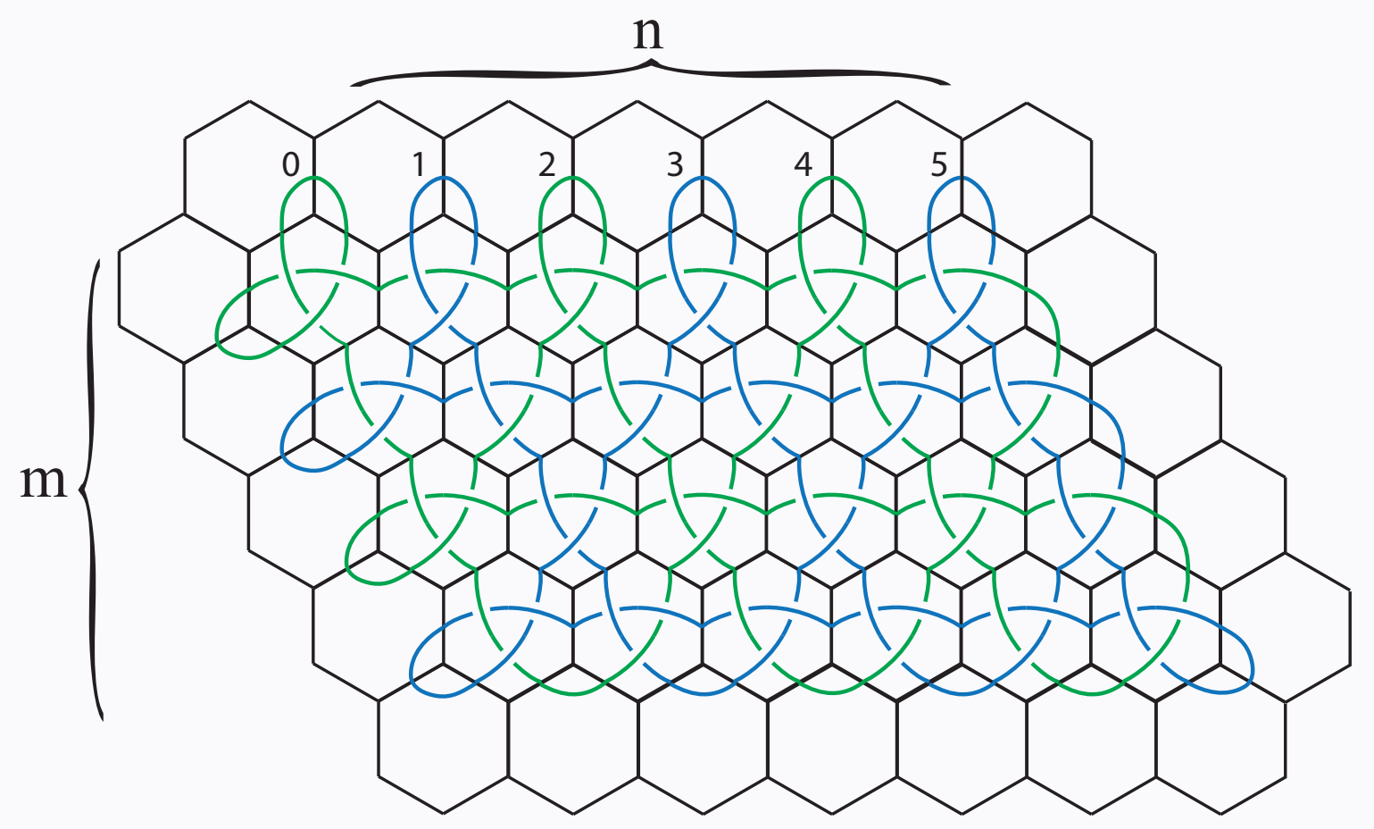


Figure 2: Stevedore Knot (6_1)

Algebraic Saturation

We saturate by placing T_{20} or T_{21} tiles in a $m \times n$ parallelogram and connecting them nontrivially, then we label the top loops with elements of $\mathbb{Z}_n$. We proved each component is represented by the distinct cosets of $\langle \gcd(m,n) \rangle$ in $\mathbb{Z}_n$



Future Work

We will continue to investigate possible invariants specific to hextiles, properties of algebraic saturation structures and composition, and prove hextile numbers for higher crossing knots and triple crossing knots.

Acknowledgements

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